ARKIV FÖR FYSIK Band 16 nr 44

Communicated 14 October 1959 by OSKAR KLEIN

On nonlocal effects in weak interactions

By M. HESSLER

With 3 figures in the text

SUMMARY

Weak interactions are assumed to be slightly nonlocal, due to the occurrence of a heavy intermediate particle. When the term for this particle is eliminated from the Lagrangian, an expression arises in which the lepton and nucleon currents interacting at different points are coupled by a Yukawa potential. The nonlocal effects in the inverse β process, $p+e^{-}\rightarrow n+\nu$, and in μ decay are calculated using V–A coupling. It is found that the value of the Michel parameter tends to increase above 0.75.

The coupling constant, g^2, used in the nonlocal interaction Hamiltonian has the dimension of charge squared in contrast to the constant of the local theory, which has the dimension of charge squared times length squared and is the one experimentally determined. By studying the relation between these two constants it is found that the experimental ϱ value which limits the characteristic length for the nonlocal effect to $\approx 10^{-13}$ cm gives a corresponding limitation to g^2 of $g^2 \geqslant 10^{-24}$ cgs units.

Introduction

According to Feynman and Gell-Mann [1] the most general form of the Universal Fermi Interaction is

$$\mathcal{H}_{\text{int}} = \sqrt{8}\, G^2 (\bar{\psi}_1 \gamma^{\mu} a \psi_2)(\bar{\psi}_3 \gamma_{\mu} a \psi_4) + \text{h.c.}, \tag{1}$$

where a is the projection operator $\frac{1}{2}(1+i\gamma_5)$, and G^2 is the coupling constant for pure Fermi transitions. G^2 has been experimentally determined from the decay of O^{14}, Al^{26} and Cl^{34}, and found to be [2] $(1.410 \pm 0.009)\cdot 10^{-49}$ erg cm^3.

If we assume that the four fermions in a weak interaction do not interact at exactly the same space-time point, the expression for $\mathcal{H}_{\text{int}}$ has to be modified. We restrict ourselves to a special kind of nonlocality, proposed by Lee and Yang [3], Tanikawa [4] and others, in which the fermions interact in pairs, while the interaction between the pairs is propagated by a heavy boson. This corresponds to a Feynman diagram of the type shown in figure 1. Phenomenologically this type of interaction can be described by a "nonlocal Hamiltonian" in which

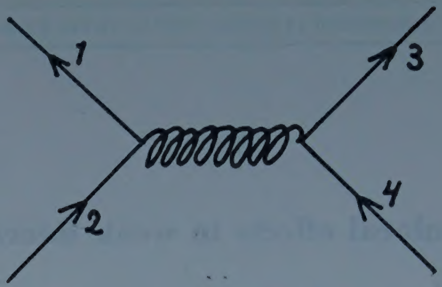

Fig. 1. A heavy boson is interchanged between fermion pairs.

the boson field has been eliminated, as has been shown by Lee and Yang [3]. The resulting interaction Hamiltonian is of the following form:

$$\mathcal{H}_{\text{int}} = \sqrt{8}\, g^2 \int [\bar{\psi}_1(x)\, \gamma^\mu a\, \psi_2(x)]\, F(\mathbf{x}-\mathbf{x}')\, [\bar{\psi}_3(x')\, \gamma_\mu a\, \psi_4(x')]\, d^3\mathbf{x}' + \text{h.c.}, \tag{2}$$

where g^2 is a new coupling constant, to be determined so as to make equation (2) give the same total probability for the process as the local form of $\mathcal{H}_{\text{int}}$. We shall specialize the function $F(\mathbf{x}-\mathbf{x}')$ to be a Yukawa potential, i.e.

$$F(\mathbf{x}-\mathbf{x}') = \frac{e^{-|\mathbf{x}-\mathbf{x}'|/l}}{|\mathbf{x}-\mathbf{x}'|}. \tag{3}$$

Here l is the length characterizing the nonlocal effects. F is related to the δ function by

$$\lim_{l\to 0} \frac{1}{4\pi l^2} F(\mathbf{x}-\mathbf{x}') = \delta(\mathbf{x}-\mathbf{x}'). \tag{4}$$

This choice gives the coupling constant the dimension of charge squared, while G^2 has the dimension of charge squared times length squared. Therefore the connexion between the constants is of the following form:

$$G^2 = K\, g^2\, l^2\, f(l), \tag{5}$$

where K is a number and f is a dimensionless function of l which tends towards 1 as l goes to zero.

Thus, if the weak interaction is somewhat nonlocal and can be described by a Yukawa potential, the constant characterizing the strength of the interaction is g^2. In order to determine g^2 from the experimentally known G^2 one has to find the magnitude of the length l. As we shall see, one effect of the nonlocality is a change in the form of the energy spectrum of the outgoing particles, which gives some information about the magnitude of l. Thus it is possible to find the value of the "true" coupling constant g^2.

In the following, this kind of nonlocal effect will be examined in two types of processes, the inverse β process, $e^- + p \to n + \nu$, where the incoming electron is assumed to have very high energy (about 1 BeV), and the μ decay $\mu \to e + \nu + \bar{\nu}$.

Inverse β process

The interaction Hamiltonian for the process $e^- + p \rightarrow n + \nu$, where the nucleon and lepton currents at two different points are supposed to interact via a heavy boson, is given by equations (2) and (3):

$$H_{\text{int}} = \sqrt{8}\, g^2 \int\!\!\int [\bar{\psi}_n (x)\, \gamma^{\mu}\, a\, \psi_p (x)] \frac{e^{-|\mathbf{x}-\mathbf{x}'|/l}}{|\mathbf{x}-\mathbf{x}'|} [\bar{\psi}_\nu (x')\, \gamma_\mu\, a\, \psi_e (x')]\, d^3\mathbf{x}'\, d^3\mathbf{x}. \tag{6}$$

The energy spectrum for the outgoing particles, calculated from this Hamiltonian for a case where protons at rest are bombarded by high energy electrons of energy E, momentum p and current density N, is

$$P(\mathbf{p}_n, \mathbf{q})\, d^3\mathbf{p}_n\, d^3\mathbf{q} = 16\,\pi^3 (4\,\pi\, l^2)^2 g^4 h^{-4} \frac{N E}{c^2 p} \left(1 - \frac{c^2}{E_n E_\nu} \mathbf{p}_n \cdot \mathbf{q}\right) \cdot$$

$$\cdot \frac{1}{(1 + l^2 \hbar^{-2} |\mathbf{p} - \mathbf{q}|^2)^2}\, \delta(\mathbf{p} - \mathbf{p}_n - \mathbf{q})\, \delta(E_i - E_f)\, d^3\mathbf{p}_n\, d^3\mathbf{q}, \tag{7}$$

where $\mathbf{p}_n$, $\mathbf{q}$ denote the momenta of neutron and neutrino respectively, and E_i, E_f are the initial and final state energies. The integration over neutron and neutrino momenta is carried out by a method due to Rau [5]. Assuming the nonlocality to be a small effect, we evaluate the expression $(1 + l^2 \hbar^{-2} |\mathbf{p} - \mathbf{q}|^2)^2$ in a series in l^2 and retain only the first term. The result is

$$P_{\text{tot}} = 16\,\pi^3 (4\,\pi\, l^2)^2 g^4 h^{-4} N \frac{\pi E B^2}{2 c^6 p^2} \left\{ \frac{2 c p}{E_0^2 - c^2 p^2} \left[1 - \frac{2 l^2}{c^2 \hbar^2} (c^2 p^2 + B) \right] + \right.$$

$$\left. + \frac{2 l^2}{c^2 \hbar^2} \left[\frac{B E_0}{2} \cdot \frac{4 E_0 c p}{(E_0^2 - c^2 p^2)^2} - \frac{B^2}{12} \cdot \frac{6 E_0 c^2 p^2 + 2 c^3 p^3}{(E_0^2 - c^2 p^2)^3} \right] \right\}, \tag{8}$$

where we use the following notations:

$$E_0 = E + M c^2, \quad \text{and } B = E_0^2 - M^2 c^4 - c^2 p^2.$$

The passage to the local limit is effected by putting $l = 0$ and changing $(4\,\pi\, l^2)\, g^2$ to G^2. To get an idea of the magnitude of the effect, we choose $E = M c^2$, i.e. $E \approx 1$ BeV. Using the experimental G^2 value of $1.41 \cdot 10^{-49}$ erg cm^3 we find the cross section equal to $6.2 \cdot 10^{-42}$ cm^2. To the first approximation in the nonlocal effect we find the cross section at the given energy

$$\sigma = 64\,\pi^4 (4\,\pi\, l^2)^2 g^4 h^{-4} M^2 \frac{1}{3} \left(1 - \frac{32}{27} \cdot \frac{l^2 M^2 c^2}{\hbar^2}\right). \tag{9}$$

Discussion

In order to be able to say something about the magnitude of l we have to go back to the energy spectrum of the neutron. This is not explicitly given when the integrations are carried through by the method used above, but it is easily seen that the nonlocality is contained in the factor $(1+l^2\hbar^{-2}p_n^2)^{-2}$. The maximum value of the neutron energy is $\approx 2\,Mc^2$. If we choose a value in the upper part of the spectrum, e.g. $E_n = 1.5\,Mc^2$, we find that the nonlocal effect amounts to 10 % if the characteristic length l is of the order of magnitude $6.6 \cdot 10^{-15}$ cm.

The calculated cross section of $6.2 \cdot 10^{-42}$ cm^2 may be compared with the cross section for the free antineutrino in the process $p+\bar{\nu}\rightarrow n+e^+$, which has been measured by Reines and Cowan [6] and found to be $(11 \pm 2.6)\cdot 10^{-44}$ cm^2. In spite of the relatively large cross section of $6 \cdot 10^{-42}$ cm^2, which is due to the high electron energy involved, it seems to be impossible at present to detect the process $p+e^-\rightarrow n+\nu$ in a scattering experiment. Such experiments with electron energies up to 550 MeV have been carried out by Hofstadter [7]. The cross sections for the various scattering phenomena, mesonic processes, etc., that occur in such experiments are of order of magnitude 10^{-25}–10^{-30} cm^2. As neutrons are produced by these processes, e.g. by electrodisintegration also, it is impossible to identify the process $p+e^-\rightarrow n+\nu$ among the others, which occur with much greater probability. The question of nonlocality is of course still more difficult to answer, so at present it is impracticable to get any information about possible nonlocal effects from experimental study of β processes.

μ decay

For the decay of the μ meson, $\mu\rightarrow e+\nu+\bar{\nu}$, we write the interaction Hamiltonian in the following way:

$$H_{\text{int}} = \sqrt{8}\,g^2 \int\!\!\int [\bar{\psi}_\nu(x)\,\gamma^\mu\, a\,\psi_\mu(x)]\,\frac{e^{-|\mathbf{x}-\mathbf{x}'|/l}}{|\mathbf{x}-\mathbf{x}'|}\,[\bar{\psi}_e(x')\,\gamma_\mu\, a\,\psi_\nu(x')]\,d^3\mathbf{x}'\,d^3\mathbf{x}, \tag{10}$$

since g^2 is supposed to be the universal coupling constant for weak interactions in the same way as G^2 is a universal constant in the local theory. The corresponding energy spectrum for the outgoing particles is

$$P(\mathbf{p},\mathbf{q}_2)\,d^3\mathbf{p}\,d^3\mathbf{q}_2 = 32\,\pi^3\,(4\,\pi\,l^2)^2\,g^4\,h^{-4}\,N\left(1-\frac{c^2}{E\,E'_\nu}\,\mathbf{q}_2\cdot\mathbf{p}\right)\cdot$$

$$\cdot\frac{1}{(1+l^2\,\hbar^{-2}\,|\mathbf{q}_1+\mathbf{p}|^2)^2}\,\delta(\mathbf{q}_1+\mathbf{q}_2+\mathbf{p})\,\delta(E_i-E_f)\,d^3\mathbf{p}\,d^3\mathbf{q}_2. \tag{11}$$

Here the following notations are used: N is the μ meson density; $\mathbf{p}$, $\mathbf{q}_1$, and $\mathbf{q}_2$ are the momenta of the electron, antineutrino, and neutrino respectively; and E, E'_ν the electron and neutrino energies.

The integration over neutrino and antineutrino momenta is carried out by the same method as was used above. Introducing the new variable x, defined by the relation

$$x = \frac{2\,c\,p}{\mu\,c^2}, \tag{12}$$

and leaving out terms containing l^2 to higher order than the first, we find the following expression for the energy spectrum of the electrons:

$$P(x) \sim x^2(3-2x) - \beta^2 x^2 (6-8x+3x^2-\tfrac{2}{5}x^3) + 0(\beta^4), \tag{13}$$

where β is a parameter related to l by $\beta = \frac{1}{2} l \hbar^{-1} \mu c$.

In the local limit $\beta = 0$ and the form of the electron spectrum is given by

$$P(x) \sim x^2(3-2x). \tag{14}$$

This is to be compared with the general expression for the energy distribution derived by Michel [8, 9] and reformulated by Sargent *et al.* [10] to give

$$P(x) \sim 4x^2[3(1-x) + \tfrac{2}{3}\varrho(4x-3)]. \tag{15}$$

Equation (14) corresponds to a ϱ value of 0.75, which is also the value given by the local V–A theory. A comparison of equations (13) and (15) shows that the nonlocal form of the spectrum cannot be described by a Michel parameter, not even in first approximation.

Much experimental work has been done on the determination of ϱ. Some of the available information is collected in table 1.

Table 1. Experimental ϱ values.

Experimentalist	ϱ value	Year	Ref.
Crowe *et al.*	0.68 ±0.02	1957	2
Dudziak and Sagane	0.736 ±0.022	1958	11
Anderson *et al.*	0.74 ±0.03	1959	12
Dudziak, Sagane and Vedder	0.741 ±0.027	1959	13
Plano and Lecourtois*	0.79 ±0.03	1958	14

In order to be able to compare these values of ϱ and the corresponding energy spectra given by equation (15) with equation (13), which includes the nonlocal effects, we must take into account the relation between the coupling constants in the local and the nonlocal theories, given by equation (5). Integration of equations (13) and (15) over all possible values of x gives the total probability for the μ decay. By equating the two expressions we find the following form for equation (5):

$$G^4 = (4\pi l^2)^2 g^4 (1 - \tfrac{16}{15}\beta^2). \tag{16}$$

Thus the coupling constant $(4\pi l^2)^2 g^4$ occurring in equation (11) has to be replaced by $G^4(1-\frac{16}{15}\beta^2)^{-1}$ when comparing the spectra for the local and the nonlocal cases. Doing so, we find that when l increases from zero to small positive values the form of the $P(x)$ curve given by equation (13) changes in the same

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* A later ϱ value found from the Plano-Lecourtois experiment is 0.80±0.02, which was reported at the International Summer School for Physics, Varenna sul Lago di Como, July 1959.

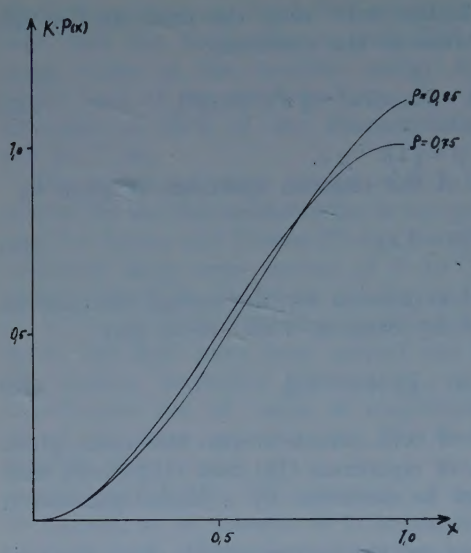

Fig. 2. $P(x)$ from equation (15).

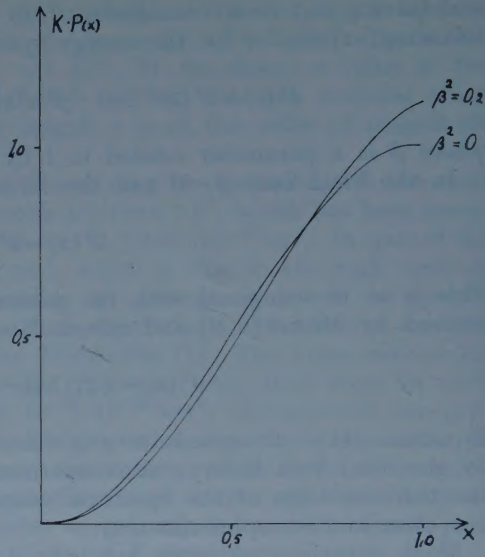

Fig. 3. $P(x)$ from equation (13).

way as does the form of $P(x)$ given by equation (15) when ϱ increases from 0.75 to higher values. This is seen from figures 2 and 3. By performing a least square fit between the two curves we find the l dependence of ϱ as indicated in table 2. This is consistent with results found by Byers and Peierls [15] and others [16, 18] from various types of two-stage models for weak interactions.

Table 2. Relation between β and ϱ.

β^2	ϱ
0.01	0.754
0.05	0.770
0.1	0.792
0.2	0.84

A ϱ value of about 0.79 corresponds to a characteristic length of $1.2 \cdot 10^{-13}$ cm and an intermediate boson of mass $M_B \approx 330\ m_e$. However, the stability of the known particles requires a heavy boson, i.e. of mass greater than that of the K meson. The lowest β^2 value in table 2 satisfies this condition, since it corresponds to a mass of $\approx 1\,000\ m_e$ and $l \approx 2 \cdot 10^{-14}$ cm. This β^2 gives a ϱ value of 0.754 which is in agreement with the experimental results of references 11–13. However the deviation from the theoretical value 0.75 is smaller than the experimental uncertainty by a factor 10, so at present the existence of a boson of this mass could not be experimentally established from the determination of ϱ. The Plano-Lecourtois result, however, requires a boson which is too light to be compatible with the stability conditions for the observed particles.

Now, from equation (16) we find the value of g^2. Leaving out the last term, as we are only dealing with magnitudes, gives

$$G^2 \approx 4\pi l^2 g^2, \tag{17}$$

which corresponds to $g^2 \approx 10^{-22}$ for $l = 10^{-14}$. The experimental ϱ values set a lower limit for g^2 of 10^{-24}. These results can be compared with the strength of the electromagnetic coupling, which is $2 \cdot 10^{-19}$ cgs units. Thus a nonlocal effect characterized by a length of 10^{-16} cm would give a coupling constant of the same order of magnitude for the weak and the electromagnetic interaction. In this case, however, the difference between the corresponding ϱ value and the theoretical one is far too small to be detected experimentally. The mass of the boson in this case is of the order of one hundred nucleon masses. This is consistent with the value 60 M found by Lopes [17] from a two-stage model for the weak interaction with the same assumption for the interaction strength.

Discussion

Since a heavy boson of the type occurring in the two-stage model for Fermi interactions would be very difficult to observe directly, any indirect information about this theory, such as that obtained from a detailed study of the spectrum form, is of interest. In addition to the μ decay discussed here there are other processes which would be affected by the existence of a heavy charged vector boson, e.g. the K_{e3} and $K_{\mu 3}$ decays, where the corresponding nonlocal effects have been calculated by Oneda and Pati [18].

It follows from what has been said above that a nonlocal Hamiltonian for weak interactions of the type given in equations (2) and (3), where the fermion pairs are coupled by a Yukawa potential, makes it possible to use a coupling constant of the same magnitude and the same dimension as the coupling constant for the electromagnetic interactions. Thus, the "weakness" of the Universal Fermi Interaction is in this theory due to its being a two-stage process.

The nonlocal effects in μ decay derived from such a Hamiltonian are consistent with most of the present experimental knowledge of this process. Future experimental work on weak interactions may give the detailed information which is necessary in order to tell more definitely whether the intermediate boson exists and, if so, how it affects the theory of the Universal Fermi Interaction.

ACKNOWLEDGEMENTS

The author wishes to express her sincere thanks to professor O. Klein, who suggested this investigation, for his kind interest and encouragement during the work. I also want to thank Dr. B. Laurent for interesting and helpful discussions, and Dr. N. Weinstein for examining the English. Further I wish to acknowledge support from the Air Research and Development Command, United States Air Force.

Institute for Theoretical Physics, University of Stockholm. October 1959.

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Tryckt den 18 februari 1960

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